Streaming, disruptive interference and power-law behavior in the exit dynamics of confined pedestrians

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Abstract

We analyze the exit dynamics of pedestrians who are initially confined in a room. Pedestrians are modeled as cellular automata and compete to escape via a known exit at the soonest possible time. A pedestrian could move forward, backward, left or right within each iteration time depending on adjacent cell vacancy and in accordance with simple rules that determine the compulsion to move and physical capability relative to his neighbors. The arching signatures of jamming were observed and the pedestrians exited in bursts of various sizes. Power-law behavior is found in the burst-size frequency distribution for exit widths \( w \) greater than one cell dimension \( (w > 1) \). The slope of the power-law curve varies with \( w \) from \(-1.3092\) \((w = 2)\) to \(-1.0720\) \((w = 20)\). Streaming which is a diffusive behavior, arises in large burst sizes and is more likely in a single-exit room with \( w = 1 \) and leads to a counterintuitive result wherein an average exit throughput \( Q \) is obtained that is higher than with \( w = 2, 3, \) or \( 4 \). For a two-exit room \( (w = 1) \), \( Q \) is not greater than twice the yield of a single-exit room. If the doors are not separated far enough \((< 4w)\), \( Q \) becomes even significantly less due to a collective slow-down that emerges among pedestrians crossing in each other’s path (disruptive interference effect). For the same \( w \) and door number, \( Q \) is also higher with relaxed pedestrians than with anxious ones.

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1. Introduction

The collective motion of many similar objects that are interacting in a well-defined manner results in a variety of phenomena [1,2] such as: jamming in traffic systems and granular materials [3–5], pedestrian behavior [6–8], flocking in birds [9], aggregate behavior in bacteria [10,11] and cluster formation in evolving networks [12]. An accurate knowledge of the collective behavior of (confined) pedestrians is valuable for it can provide new insights in the formulation of more effective strategies of managing crowd escape in a crowded hall or stadium as well as the design of efficient exits in small rooms.

Several approaches have been used to study the collective behavior in pedestrians such as modeling them as gas (particle)- or liquid (fluid)-analog [13]. Helbing et al. utilized a generalized force model to investigate panic and jamming by uncoordinated motion in crowds [6]. Cellular automata (CA) are also used in traffic simulations enabling simplifications in the model [3,14]. The CA model is a compromise between the microscopic approach, which treats the pedestrians as interacting particles, and the macroscopic model which views them as a (continuous) fluid flow. The CA model is particularly attractive for its conceptual simplicity and numerical efficiency.

In this paper, we determine the exit dynamics of pedestrians who are initially confined inside a two-dimensional room. The pedestrians who are modeled as CA compete with each other to escape via a known exit at the earliest possible time. A pedestrian could move forward, backward, left or right within each iteration time unit depending on adjacent cell vacancy and in accordance to simple rules that determine their compulsion to move and physical capability relative to his/her neighbors. We investigate the characteristics of the exit rate and the throughput (total number of pedestrians that has escaped within a given duration of time) as a function of exit width (door size $w$) and door separation. We search for possible power-law relations that indicate self-organized criticality and avalanche behavior in the exit dynamics [15,16].

The rest of the paper is organized as follows: Section 2 describes our CA model and the simulation procedures. The simulation results are presented in Section 3 and discussed in Section 4. Our simulation has reproduced the arching behavior near the exit [4,6,13]. A number of counterintuitive characteristics were also seen in the throughput dependence with exit width and exit number. For a single-exit room, queuing was more likely to emerge spontaneously with the smallest possible door width of $w = 1$ than with larger ones. Pedestrians were also found to escape in bursts of different sizes at larger door widths.

2. Pedestrians as cellular automata

A two-dimensional CA is used to simulate the movement and exit of pedestrians in a room that is divided into $P \times Q$ cells where each cell is either empty (0) or occupied (1) by no more than one pedestrian at a time. Initially (at iteration time $k = 0$), pedestrians are distributed randomly. The room exit is $w$-cells wide so that at most $w$ pedestrians can escape from the room simultaneously.
Fig. 1. Pedestrian $P_i$ as a cellular automaton. $P_i$ faces the empty cell that is nearest to the exit. He occupies the cell if $R + L < B + \phi$, where $\phi$ is a measure of his eagerness to move. $R$, $L$, and $B$ are the total number of neighbors to his right ($r$), left ($l$) and back ($b$), respectively. Other pedestrians who are separated from $P_i$ by a vacant cell are not counted as neighbors.

The exit location is known beforehand to all pedestrians who compete to leave the room at the earliest possible time. The $i$th pedestrian $P_i$ escapes by occupying a neighboring empty cell along the path towards the exit. If the said preferred cell is occupied then he randomly tries to move to one of the other adjacent empty cells with a uniform probability distribution.

Pedestrian $P_i$ faces the vacant cell along the exit path and moves if $L + R < B + \phi$ where $\phi$ is a measure of his level of anxiety or panic, otherwise he stays. $L$, $R$, and $B$ represents the total number of neighbors to the left ($l$), right ($r$) and back ($b$) directions of $P_i$, respectively (see Fig. 1). $P_i$ is considered as having escaped from the room upon occupying the exit. Per iteration step $k$, the process is repeated for each of the pedestrians inside the room.

3. Experimental results

3.1. Arching

First, we test our model for the presence of arching which is a signature of jamming in granular materials [4]. Arching indicates a jammed state that restricts considerably
the ability of pedestrians to escape from the room. The dynamics of pedestrians leaving through an exit was simulated for a room (18 × 14 cells) that initially (at k = 0) contains 200 randomly distributed pedestrians (ϕ = 5). At k = 0, the pedestrians and the exit door (marked by a circle) are purposely separated by two rows of unfilled cells to prevent the premature occurrence of jamming.

Figs. 2B–D illustrate three occasions of arching that emerge near the exit from an initial random configuration (2A) with ϕ = 5 and k = 100. The arching in Fig. 2B was formed in a room with a single-exit door of width w = 1. Fig. 2C shows the arch that arises with a larger door width (w = 2). The arching distribution has a larger radius but more vacant cells are found within its perimeter. Note, however, that the number of pedestrians who remained in the room after more than 100 iterations is different for w = 1 and 2.

Fig. 2D illustrates that two arches are formed when the room has two-exit doors each of size w = 1. The arches are each centered on an exit. Separating the doors farther apart eventually leads to non-overlapping arches where the average throughput Q which we define as the average total number of pedestrians that has exited per unit time is twice the value yielded by a single-exit room with w = 1.

3.2. Exit throughput

We investigate the throughput behavior under different exit conditions for a room with a size of 18 × 14 cells. A new pedestrian is added into the room at a randomly
chosen empty cell every time a pedestrian leaves the exit. This is done to ensure a long-term ensemble behavior for the system. At any given time, the room contains 30 competing pedestrians (11.9% of room capacity) and \( Q \) is calculated after \( 5 \times 10^4 \) iterations (\( k = 5 \times 10^4 \)).

Fig. 3 plots average throughput \( Q \) as a function of \( w \) for a single-exit room over 10 independent trials. The simulation results are highly reproducible as indicated by the very low values of the associated standard deviations. For \( w \geq 2 \), a linear growth in \( Q \) is found with increasing \( w \). However, a counterintuitive result is observed with \( w = 1 \) where the corresponding \( Q \) is higher than those obtained at larger \( w \) values. For \( 1 < w \leq 4 \), the \( Q \) values never exceed that obtained with \( w = 1 \). Larger \( Q \) values are obtained only with \( w > 4 \). The smaller \( Q \) values occur because of jamming near the exit which disrupts the streaming of escaping pedestrians that is more likely to occur with \( w = 1 \). Although arching is also present around an exit with \( w = 1 \), the limited door width is conducive to the spontaneous queue formation that favors the exit of one pedestrian per iteration step thereby creating a continuous (efficient) stream of escaping pedestrians over long time periods. Gaps may occur in the queue but such inefficiencies are relatively rare. In the absence of gaps, streaming over a time duration of \( 5 \times 10^4 \) iterations yields a throughput of \( Q \approx 1 \).

With larger door widths, a pedestrian is confronted with more choices representing a larger number of possible trajectories towards a chosen exit. These trajectories could impede the movement of the other competing pedestrians resulting in more empty (unfilled) cells arising particularly near the exit (see Fig. 2C). We point out that the results replicate the behavior exhibited by a crowd in a state of panic where an
Fig. 4. Dependence of $Q$ with $d$ for a two-exit room (size: 18 × 14 cells, $\varphi = 5$, $w = 1$) after $k = 5 \times 10^4$. The solid line represents the throughput that is yielded by a single-exit room with $w = 1$.

individual instinctively prefers to go to the exit location he last remembered in high disregard of other (more efficient) escape possibilities. At large enough $w$ values, more exit space becomes available and $Q$ increases linearly with $w$ as expected.

Fig. 4 plots $Q$ as a function of the door separation $d$ in a two-exit room with $w = 1$ for each door. $Q$ increases linearly with $d$ until $d = 4$, after which it saturates to a value near $Q = 1.6$. Separating the two exits by more than $d = 4$ does not result in a significant $Q$ increase for a room that is filled at 11.9% of capacity at all times. At $d > 4$, two non-overlapping arches are formed and the exit behavior of each arch becomes independent of the other for given $w$ and $\varphi$ values.

Interestingly, the saturation value of $Q \approx 1.6$ is less than twice the $Q$ value of 0.912294 that is yielded by a single-exit room with $w = 1$ and $\varphi = 5$ (see Fig. 3). Doubling the number of exits in the room does not automatically double the $Q$ value because of disruptive interference effects. Furthermore, two separate doors are better than a single door ($d = 0$) of the same total width. A similar result has been earlier observed [8] for pedestrian counter streams.

Quiescent times exist when jamming prevents a pedestrian from reaching the exit. Pedestrians escape from the room in bursts of various sizes $S$. For $w = 1$, the queue that emerged translates to a relatively continuous stream (large $S$ values) of exiting pedestrians. Fig. 5 plots burst-size frequency distribution $F(S)$ for various $w$ values. We search for possible power-law relation that would imply the presence of self-organized criticality [15,16].

For $w = 1$ (squares), an exponential decay with increasing $S$ is found for $F(S)$-large bursts are much less likely to happen than smaller ones. However, a power-law
behavior is observed for small $S$ values at $w \geq 2$. The same behavior also seems likely for the entire $S$-range if longer observation times are used. In Fig. 5A, the constants in the power-law equation: $F = F_0 S^z$, assume the following values at different $w$ values: (a) $F_0 = 13,355$, $z = -1.53$, $w = 2$; (b) $F_0 = 17,178$, $z = -1.33$, $w = 3$; and (c) $F_0 = 51,522$, $z = -1.28$, $w = 4$. 

Fig. 5. Burst-size frequency distribution $F(S)$ for $w = 1$ (squares), 2 (crosshairs), 3 (circles), 4 (solid circles), 10 (diamond), 20 (solid diamond) where $\phi = 5$, and $k = 5 \times 10^4$: (A) $18 \times 14$ cell room and (B) $38 \times 30$ cell room. In Fig. 5(A), the solid line is described by $F(S) = 202,388 \exp(-0.511S)$. In Fig. 5(B), the solid line is described by $F(S) = 31,464 \exp(-0.4682S)$. Power-law behavior is exhibited only for $w \geq 2$. Both rooms have the same occupancy rate of about 12%.
Fig. 6. Burst-size frequency distribution $F(S)$ for different $\varphi$ for an $18 \times 14$ cell room, $k = 5 \times 10^4$ and $w = 2$. The $F(S)$ distributions peaks at around $S = 4$ regardless of $\varphi$.

Fig. 5B shows a similar power-law behavior for a larger system with $N = 135$ and room size of $38 \times 30$ cells. The constants in the power-law equation: $F = F_0 S^2$, assume the following values at different door sizes: (a) $F_0 = 1026.8$, $\alpha = -1.3092$, $w = 2$; (b) $F_0 = 1098$, $\alpha = -1.2198$, $w = 3$; (c) $F_0 = 2321$, $\alpha = -1.1699$, $w = 4$; (d) $F_0 = 4393.9$, $\alpha = -1.1517$, $w = 10$; (e) $F_0 = 6339.1$, $\alpha = -1.0720$, $w = 20$. Because the value of $\varphi = 5$ is maintained for the larger system, $F$ decreases due to the response of the movement rule ($L + R < B + \varphi$) to an increased number of pedestrians in the room. Complete jamming (i.e., no exiting pedestrian) is observed at $k \sim 5000$.

Fig. 6 plots $F(S)$ at different $\varphi$ values for a single-exit room with $w = 2$. The average throughput $Q$ which is given by the area under the $F(S)$ curve, decreases with increasing $\varphi$. For a room filled with highly agitated pedestrians with extreme tendencies to move (high $\varphi$), $Q$ is smaller than that obtained with a room of relaxed pedestrians (low $\varphi$) who tend to leave the room in an orderly (streaming) manner. Fig. 6 reveals a most probable burst-size value of $S = 4$ that is independent of $\varphi$. A power-law fit to the peak $F(S = 4)$ values with $\varphi$ yields $F(4) = 337.184 \varphi^{-1.145}$. Streaming is much less likely with agitated individuals.

Using a continuous model with a generalized force equation Helbing et al. [6] also observed a similar phenomenon that they called the “faster-is-slower” effect that is associated to the impatience of anxious pedestrians. We found the same results with a simple and computationally efficient CA model that utilizes the parameter $\varphi$ as an analogue to Helbing’s panic parameter.
4. Discussion

The CA model succeeded in showing the arching behavior of crowds around the exit of a room (Fig. 2). Fig. 4 reveals that, for a two-exit room, the throughput never exceeds $Q = 2$. If the doors are not separated far enough ($d < 4w$), $Q$ is lessened by a collective slow-down among pedestrians who are crossing in each other’s path. Experiments indicate that the maximum throughput that is yielded by an $N$-exit room ($w = 1$) stays noticeably lower than $NQ(w = 1)$. For a given room size, increasing $N$ does not necessarily lead to a proportional increase in $Q$—as the doors become nearer to each other disruptive interference is more likely between pedestrians and a non-linear behavior between $Q$ and $d$ arises.

Pedestrians escape from the room in bursts of various sizes $S$. In a single-exit room streaming is most likely with $w = 1$ that permits the escape of only one pedestrian per unit time. For $w > 1$, a power-law behavior was found for the burst-size frequency distribution $F(S)$. The power-law behavior is established over an $S$-range that is two orders of magnitude wide for $w \geq 20$. The bursts phenomenon in the exiting pedestrian population is akin to the avalanches that were found in sand piles [15,17]. Such critical transitions are exhibited only when streaming is less likely to occur.

Interest in exit dynamics goes beyond pedestrian behavior at the macroscopic scale. The transport of molecules through the cellular membrane is comparable to the pedestrians analyzed in this study. Large proteins are shuttled through back-and-forth pores in the cell membrane [18] and even the nucleus [19]. Better understanding of the transport properties of discrete interacting components in a complex system may be acquired through the use of strip-down (but computationally efficient) models such as the CA model utilized in this work.

5. Conclusion

We have used the computationally efficient CA model to study the exit dynamics of pedestrians in a room. The model could reproduce the classical signature of jamming—the formation of arches at the exits. A number of interesting properties such as streaming and disruptive interference were found in the exit throughput. For rooms with exit door widths that could accommodate the simultaneous exit of more than one pedestrian at any given time, exiting pedestrians leave the room in bursts of different sizes. A power-law behavior is observed for the $F(S)$ plots over an $S$-range that is at least two orders of magnitude wide for a sufficiently large single-exit room with $w \geq 20$ which is a possible indication of self-organized critical behavior.

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